HW 8
Electronic Communication Systems
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California State University, Fullerson

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Contents

1 Questions

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Drill 3.4)

$$V_2 = a_1 v_1(t) + a_2 v_1^2(t) \qquad (1)$$

where, $v_1(t) = A_c \cos 2\pi f_c t + m(t)$ (2)

Subst. eq. (2) into eq (1)

$$v_2(t) = a_1\left[A_c \cos 2\pi f_c t + m(t)\right] + a_2\left[A_c \cos 2\pi f_c t + m(t)\right]^2$$

$$\Rightarrow v_2(t) = \underbrace{a_1 A_c\left[1 + \frac{2a_2}{a_1} m(t)\right]\cos 2\pi f_c t}_{\text{AM Signal}} + a_1 m(t) + a_2 m^2(t) + a_2 A_c^2 \underbrace{\cos^2(2\pi f_c t)}_{\frac{1}{2}[1+\cos 4\pi f_c t]}$$

The signal at the output of bandpass filter is:

$$v_0(t) = a_1 A_M\left[1 + \frac{2a_2}{a_1} m(t)\right]\cos 2\pi f_c t$$

Which is an AM wave.

2 Key solution

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Drill Prob. #3.4) $V_2(t) = a_1 V_1(t) + a_2 V_1^2(t)$ (1) , $V_1(t) = A_c\cos(2\pi f_c t) + m(t)$ (2)

$$V_2(t) = a_1 A_c \cos 2\pi f_c t + a_1 m(t) + a_2 A_c^2 \cos^2 2\pi f_c t + a_2 m^2(t) + 2 a_2 A_c m(t) \cos 2\pi f_c t \quad (3)$$

Assume $M(f)$ has the following form:

fig #.

Set $g(t) \triangleq m^2(t) = m(t) \cdot m(t)$

$\Rightarrow G(f) = M(f) \otimes M(f)$ The spectrum of $g(t) = m^2(t)$ will extend from $-2W$ to $2W$ Hz, for example would be

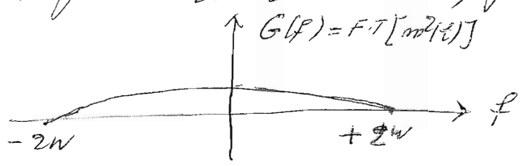

fig #2

Note: If you want to ~~g(t)~~ find $G(f)$, then you have to do $G(f) = \int_{-\infty}^{+\infty} M(\lambda) M(f-\lambda) d\lambda$. We are not interested to find the exact equation of $G(f)$, all we need to know is that the spectrum of $G(f) = F.T[m^2(t)]$ will extend from $-2W$ to $2W$ Hz.

Let us to take the F.T of eq (3) and plot it!

$$V_2(t) = a_1 A_c \cos 2\pi f_c t + a_1 m(t) + \frac{a_2 A_c^2}{2}\left[1 + \cos 4\pi f_c t\right] + a_2 m^2(t) + 2 a_2 A_c m(t) \cos 2\pi f_c t \quad (4)$$

$$\Rightarrow V_2(f) = \frac{a_1 A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + a_1 M(f) + \frac{a_2 A_c^2}{2}\delta(f) + \frac{a_2 A_c^2}{4}\left[\delta(f-2f_c) + \delta(f+2f_c)\right] + a_2 \underbrace{F.T[m^2(t)]}_{G(f)} + a_2 A_c\left[M(f-f_c) + M(f+f_c)\right] \quad (5)$$

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The plot of eq.(5) is shown in figure # 3.

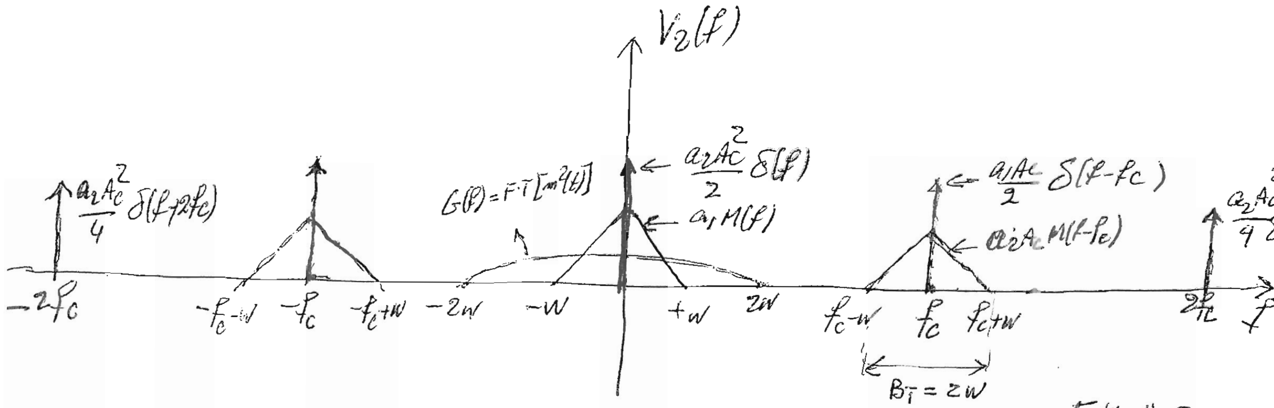

Fig # 3

Fig # 3 shows the spectral content of $V_2(t)$.

c) To extract the desired AM signal, use eq (4) and identify the AM signal:

$$V_2(t) = \underbrace{a_1 A_c\left[1 + \frac{2a_2}{a_1} m(t)\right]\cos 2\pi f_c t}_{\text{Desired A.M Signal}} + \underbrace{\left(a_1 m(t) + a_2 m^2(t) + \frac{a_2 A_c^2}{2} + \frac{a_2 A_c^2}{2}\cos 4\pi f_c t\right)}_{\text{undesired Component}} \quad (6)$$

A Bandpass filter centered at f_c with total extend of $2W$ Hz, That is having a transfer function of:

$$H(f) = \text{rect}\left(\frac{f-f_c}{2W}\right) + \text{rec}\left(\frac{f+f_c}{2W}\right) \qquad (7)$$

will pass the desired signal (AM signal) and eliminated the unwanted components.

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Using eq. (7) and figure #(3) we see that the required B.P.F must have a bandwidth of 2W Hz and centered at f_c, thus the cut-off frequencies of BPF are $f_c - W$ and $f_c + W$ Hz.

c) To avoide spectral overlapping of the desired signal (AM signal) with that of unwanted signals in $v_2(t)$, using figure # 3, we see that

1) $f_c - W \geq 2W \Rightarrow f_c \geq 3W$
2) $f_c + W \leq 2f_c \Rightarrow f_c \geq W$

Thus $f_c > 3W$

3.23

Assume $m(t)$ with spectrum of

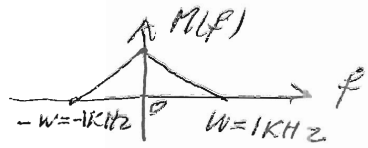
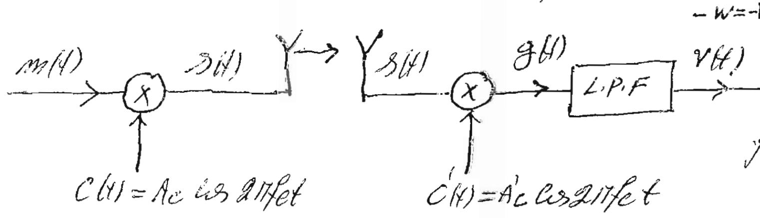


fig #1



fig#(2) Coherent detection of DSB-SC.

$s(t) = m(t)\,c(t) = A_c m(t) \cos 2\pi f_c t \Rightarrow S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$ (1)

$g(t) = s(t)\cdot c'(t) = A_c A'_c m(t) \cos^2 2\pi f_c t = \frac{A_c A'_c}{2} m(t)\left[1 + \cos 4\pi f_c t\right]$ (2)

$G(f) = \frac{A_c A'_c}{2} M(f) + \frac{A_c A'_c}{4}\left[M(f-2f_c) + M(f+2f_c)\right]$ (3)

a) For $f_c = 1.25$ KHz, the spectrum of $m(t)$, the spectrum of $s(t)$ and the spectrum of $v(t)$ (detector output) are:

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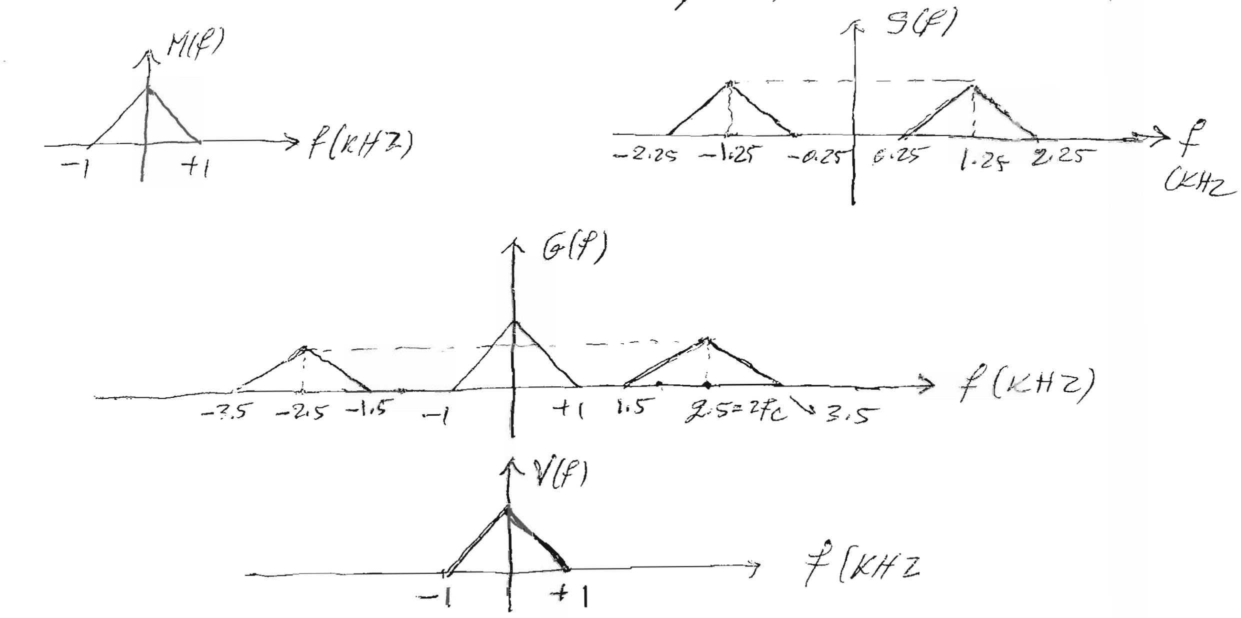
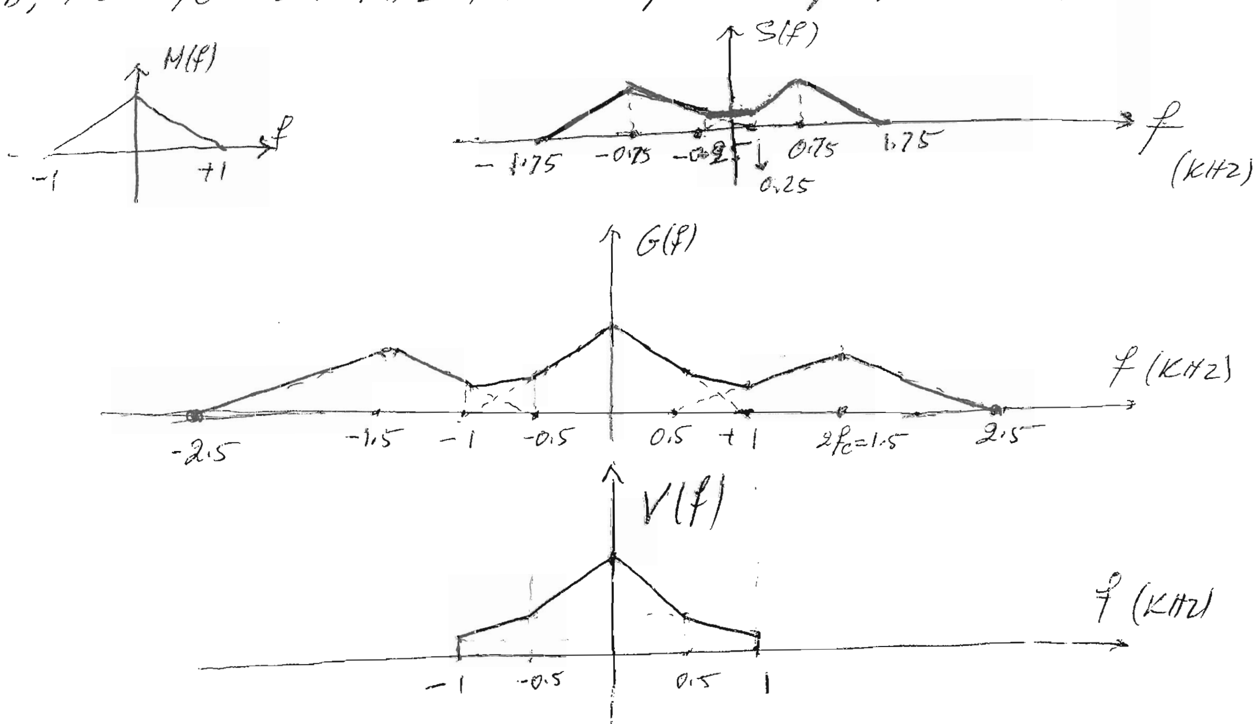


b) For $f_c = 0.75$ KHZ, the respective spectra are :



3 my graded HW

HW # 8

BE 443

Fall 2008
CSUF.

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18/20

Problem 5-1

AM broadcast transmitter is tested by feeding RF output into 50 Ω load. Tone Modulation is used. Carrier frequency is 850 KHz and power output is 5000 W. The sinusoidal tone of 1000 Hz is set for 90% modulation.

a) Evaluate the FCC Power in dBK (dB over 1 KW) units.

b) Write an equation for the voltage that appears across the 50-Ω load, giving numerical values for all constants.

c) Sketch the spectrum of this voltage as it would appear on a calibrated spectrum analyzer.

d) What is the average power that is being dissipated in the dummy load?

e) What is peak envelope power?

Answer

a) $10 \log_{10}\left(\frac{5000}{1000}\right) = 6.9897 \sim \boxed{7 \text{ dbK}}$

b) $s(t) = A_c(1+\mu \cos \omega_m t) \cos \omega_c t$ —— (1)

where ω_m is the tone frequency $2\pi(1000)$ rad/sec.
and ω_c is the carrier frequency $2\pi(850{,}000)$ rad/sec.
$\mu = .9$. Need to find A_c:

Carrier Power $= \frac{A_c^2}{2}$. but this is normalized to $1\,\Omega$.

hence $\boxed{P = \left(\frac{A_c^2}{2}\right)\frac{1}{R}}$ where $R = 50\,\Omega$.

So $P = \frac{A_c^2}{100} = \quad A_c = \sqrt{100(P)}$, but $P = 5000$ Watt FCC

So $A_c = \sqrt{100(5000)} = \boxed{707.1 \text{ V}}$

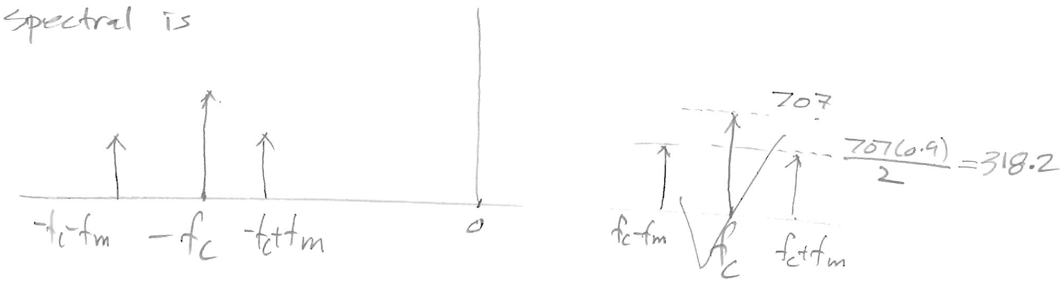
So Voltage equation is (1) given by

$$\boxed{s(t) = 707\left(1 + 0.9 \cos(2\pi \times 1000)t\right) \cos(2\pi \times 850{,}000)t}$$

→

$$s(t) = 707\cos 2\pi f_c t + \frac{707(0.9)}{2}\left[\cos(2\pi(f_c+f_m)t + \cos 2\pi(f_c-f_m)t\right]$$

hence spectral is



where $f_c = 850\,KHZ$
$f_m = 1\,KHZ$

so $f_c+f_m = 851\,KHz$
$f_c-f_m = 849\,KHz$

(d) The total Normalized average power $= \underbrace{\left(\frac{A_c^2}{2}\right)}_{\text{carrier}} + \underbrace{\left(\frac{A_c\mu}{2}\right)^2}_{\text{both sidebands.}}$

hence, we $R=50$, we obtain

$$P_{av.}\text{ in }R = \frac{1}{R}\left[\frac{A_c^2}{2} + \frac{A_c^2\mu^2}{4}\right]$$

$$= \frac{1}{50}\left[\frac{707^2}{2} + \frac{707^2(.9)^2}{4}\right] = \boxed{7{,}022.87\text{ Watt}}$$

(e) $A_{max} = A_c(1+\mu)$

hence Peak Power (average) is $\frac{[A_c(1+\mu)]^2}{2} \times \frac{1}{R}$

$$= \frac{(707(1+0.9))^2}{2\times 50} = \boxed{18{,}044.5\text{ Watt}}$$ this's not Normalized.

Normalized PEP $= \frac{[707(1+.9)]^2}{2} = \boxed{902{,}227\text{ watt}}$

5-3

AM transmitter modulated with $m(t) = 0.2\sin W_1 t + 0.5\cos W_2 t$
$f_1 = 500\,HZ$, $f_2 = 500\sqrt{2}\,HZ$. $A_c = 100$.

(a) Evaluate average power of the AM signal
(b) Evaluate Peak Envelope Power (PEP).

Answer

(a) average power (normalized) is given by

$$\frac{A_c^2}{2} + \left(\frac{A_c \mu}{2}\right)^2$$

$s(t) = A_c(1 + 0.2\sin W_1 t + .5\cos W_2 t)\cos W_c t$ ⇒ expands 2 sinusoid f.
⇒ another 2 sin. f.

$= A_c \cos W_c t + .2 A_c \sin W_1 t + .5 A_c \cos W_2 t$

X.

so normalized average power $\frac{A_c^2}{2} + \frac{(.2A_c)^2}{2} + \frac{(.5A_c)^2}{2}$

$$= \frac{100^2}{2} + \frac{20^2}{2} + \frac{50^2}{2} = \boxed{6{,}450 \text{ Watt}}$$

X . See sol.

When using Load $R = 50\Omega$ given in problem 5-2, we obtain

$$\frac{6450}{50} = \boxed{129 \text{ Watt}}$$

✓

(b) $A_{max} = A_c(1+\mu)$

hence $PEP = \frac{[100(1+.2)]^2}{2} + \frac{[100(1+.5)]^2}{\cancel{2}} =$

$= 7{,}200 + 11{,}250 = \boxed{18{,}45\cancel{0} \text{ Watt}}$

✓

PEP over load of 50Ω, we obtain

$$PEP = \frac{[100(1.2)]^2}{2\times 50} + \frac{[100(1.5)]^2}{2\times 50} = \boxed{369 \text{ Watt}}.$$

5-7

A DSB-SC signal is modulated by $m(t) = \cos\omega_1 t + 2\cos 2\omega_1 t$

where $f_1 = 500\,\text{Hz}$ and $A_c = 1$.

(a) write expression for DSB-SC signal and sketch picture of this waveform

(b) Evaluate and sketch the spectrum of this signal.

(c) Find the average (normalized) power

d) Find PEP (normalized).

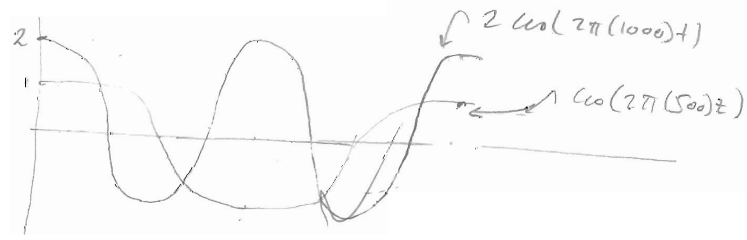
Answer

DSB-SC is double sided carrier suppressed.

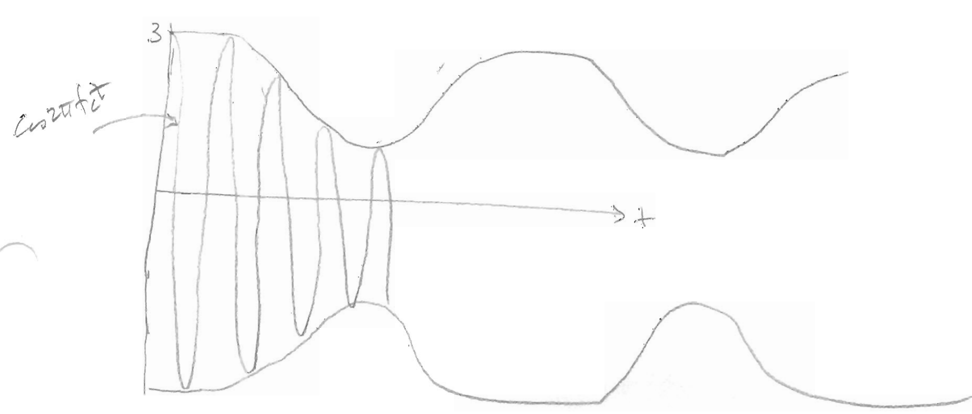
a) $\boxed{s(t) = A_c \cos\omega_c t \; m(t)}$

hence $s(t) = \cos 2\pi f_c t \left(\cos\omega_1 t + 2\cos 2\omega_1 t\right)$

$$= \cos 2\pi f_c t \, \underbrace{\left(\cos 2\pi(500)t + 2\cos(2\pi(1000)t)\right)}_{a(t)}$$



add these 2 signals ⇒ and multiply by carrier



→

b) $$s(t) = \cos 2\pi f_c \left(\cos 2\pi f_1 t + 2\cos 2\pi 2 f_1 t \right)$$

$$= \cos 2\pi f_c \cos 2\pi f_1 t + 2\cos 2\pi f_c \cos 4\pi f_1 t$$

$$= \frac{1}{2}\left(\cos 2\pi (f_c+f_1)t + \cos(2\pi(f_c-f_1)t) \right)$$

$$+ \left(\cos 2\pi (f_c+2f_1)t + \cos(2\pi(f_c-2f_1)t) \right) \text{ --- (1)}$$

so $$S(f) = \frac{1}{2}\cdot\Big[\frac{1}{2}\left(\delta(f+f_c+f_1) + \delta(f-f_c-f_1)\right)$$

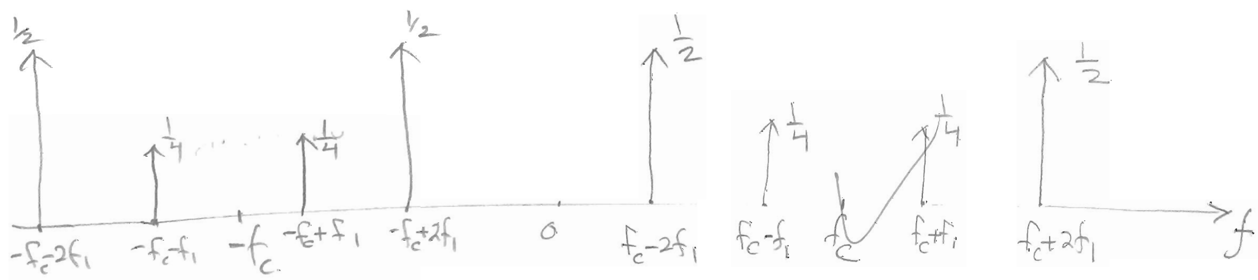
$$+ \frac{1}{2}\left(\delta(f+f_c-f_1) + \delta(f-f_c+f_1)\right)\Big]$$

$$+ \frac{1}{2}\left(\delta(f+f_c+2f_1) + \delta(f-f_c-2f_1)\right)$$

$$+ \frac{1}{2}\left(\delta(f+f_c-2f_1) + \delta(f-f_c+2f_1)\right)$$

$$= \frac{1}{4}\delta(f+f_c+f_1) + \frac{1}{4}\delta(f-f_c-f_1) + \frac{1}{4}\delta(f+f_c-f_1) + \frac{1}{4}\delta(f-f_c+f_1)$$

$$+ \frac{1}{2}\delta(f+f_c+2f_1) + \frac{1}{2}\delta(f-f_c-2f_1) + \frac{1}{2}\delta(f+f_c-2f_1) + \frac{1}{2}\delta(f-f_c+2f_1)$$



c) average power: From (1) using Formula $\frac{\text{Amplitude}^2}{2}$ per tone:

$$\frac{0.5^2}{2} + \frac{0.5^2}{2} + \frac{1^2}{2} + \frac{1^2}{2} = \frac{1}{8} + \frac{1}{8} + \frac{1}{2} + \frac{1}{2} = \boxed{1.25 \text{ Watt}}$$

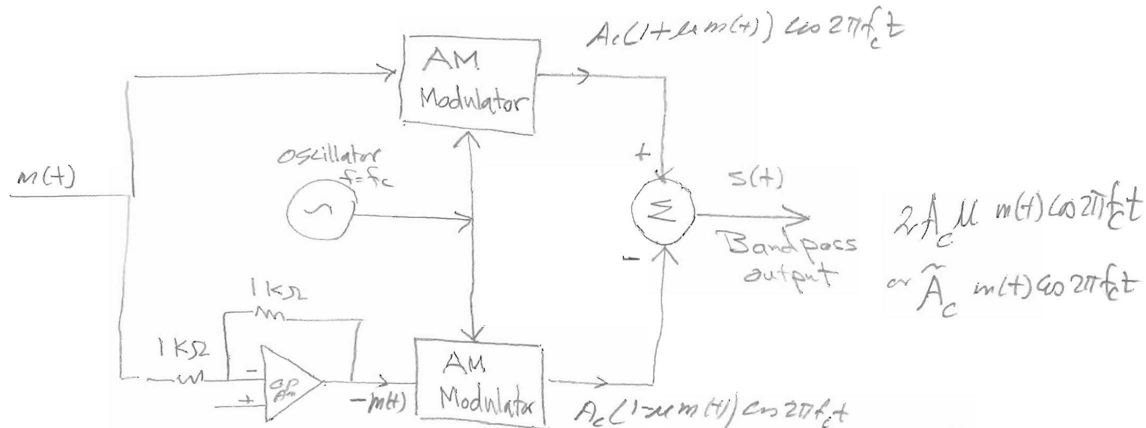
d) $A_{max} = A_c + \mu$. $\mu = 2$ for the signal $\cos\omega_1 t \cdot 2\cos 2\omega_1 t$

hence $A_{max} = 1 + 2 = \boxed{3}$

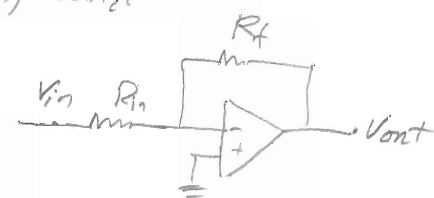
so $$PEP = \frac{A_{max}^2}{2} = \frac{9}{2} = \boxed{4.5 \text{ watt}}$$

5-9 A DSB-SC signal can be generated from 2 AM signals. Using mathematics to describe signals at each point on figure, prove output is DSB-SC.

Answer



op-Am acts as an inverting Amplifier



$$\boxed{V_{out} = -V_{in}\left(\frac{R_f}{R_{in}}\right)}$$

since $R_f = R_{in}$ in this problem, then $V_{out} = -V_{in}$.

hence $s(t) = A_c(1+\mu m(t))\cos 2\pi f_c t - [A_c(1-\mu m(t))\cos 2\pi f_c t]$

or

$$s(t) = \cancel{A_c\cos 2\pi f_c t} + A_c\mu m(t)\cos 2\pi f_c t - [\cancel{A_c\cos 2\pi f_c t} - A_c\mu m(t)\cos 2\pi f_c t]$$

$$= 2A_c\mu m(t)\cos 2\pi f_c t$$

Combine $2A_c\mu \Rightarrow \tilde{A}_c$ we obtain equation for DSB-SC

$$\boxed{\tilde{A}_c\, m(t)\cos 2\pi f_c t}$$

hence the above circuit suppresses the carrier part.

8.50 From Text Book

Find spectral density $S_Z(f)$ if:

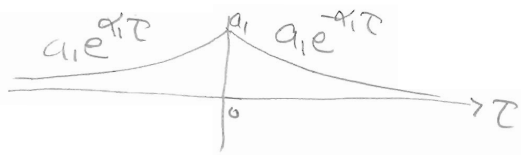
$$Z(t) = X(t) + Y(t)$$

Where $X(t), Y(t)$ are independent zero-mean R.P. with

$$R_X(\tau) = a_1 e^{-\alpha_1|\tau|} \quad \text{and} \quad R_Y(\tau) = a_2 e^{-\alpha_2|\tau|}$$

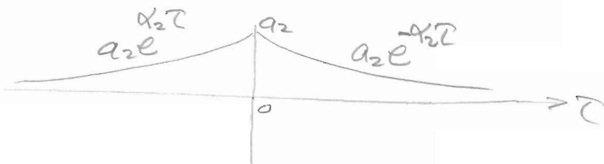
Answer

$R_X(\tau) =$



$\alpha_1 > 0$

$R_Y(\tau)$:



$\alpha_2 > 0$

The following are possible ways to solve this problem:

1. Find $R_Z(\tau)$ by adding $R_X(\tau) + R_Y(\tau)$. Then Find Fourier Transform of $R_Z(\tau)$, this gives $S_Z(f)$.
2. Find Fourier Transform of $R_X(\tau)$ and $R_Y(\tau)$. This gives $S_X(f)$ and $S_Y(f)$. Then due to Linearity of Fourier transform, add $S_X(f) + S_Y(f)$ to obtain $S_Z(f)$.

using method ① . First need to show that $R_Z(\tau) = R_X(\tau) + R_Y(\tau)$:

$$R_Z(\tau) = E\left\{ \left(X(t) + Y(t)\right)\left(X(t+\tau) + Y(t+\tau)\right) \right\}$$

$$= E\left\{ X(t)X(t+\tau) + X(t)Y(t+\tau) + Y(t)X(t+\tau) + Y(t)Y(t+\tau) \right\}$$

$$= R_X(\tau) + E\left(X(t)Y(t+\tau)\right) + E\left(Y(t)X(t+\tau)\right) + R_Y(\tau)$$

since indep.

$$\underbrace{E(X(t))}_{=0}\underbrace{E(Y(t+\tau))}_{=0} + \underbrace{E(Y(t))}_{=0}\underbrace{E(X(t+\tau))}_{=0}$$

so $\boxed{R_Z(\tau) = R_X(\tau) + R_Y(\tau)}$

hence $R_Z(\tau) = \left(a_1 e^{\alpha_1 \tau} + a_2 e^{\alpha_2 \tau}\right) u(-\tau) + \left(a_1 e^{-\alpha_1 \tau} + a_2 e^{-\alpha_2 \tau}\right) u(t)$ →

[illegible] $\int_{-\infty} a_1 e$ [illegible] $d\tau = a_1 \int_{-\infty}$ [illegible] $= a_1$ [illegible] $\frac{[illegible]}{-j2\pi f+\alpha_1}$ [illegible]

$$= \frac{a_1}{-j2\pi f+\alpha_1}\left(1\right) = \boxed{\frac{a_1}{-j2\pi f+\alpha_1}}$$

and $\int_{-\infty}^{0} a_2 e^{\alpha_2\tau} e^{-j2\pi f\tau}\, d\tau = \boxed{\frac{a_2}{-j2\pi f+\alpha_2}}$

and $\int_{0}^{\infty} a_1 e^{-\alpha_1\tau} e^{-j2\pi f\tau}\, d\tau = a_1 \int_{0}^{\infty} e^{\tau(-\alpha_1-j2\pi f)}\, d\tau$

$$= a_1 \frac{\left[e^{\tau(-\alpha_1-j2\pi f)}\right]_0^{\infty}}{-\alpha_1-j2\pi f} = \frac{a_1}{-\alpha_1-j2\pi f}(-1) = \boxed{\frac{a_1}{\alpha_1+j2\pi f}}$$

and $\int_{0}^{\infty} a_2 e^{-\alpha_2\tau} e^{-j2\pi f\tau}\, d\tau = \boxed{\frac{a_2}{\alpha_2+j2\pi f}}$

hence $\boxed{S_Z(f) = \frac{a_1}{\alpha_1-j2\pi f} + \frac{a_2}{\alpha_2-j2\pi f} + \frac{a_1}{\alpha_1+j2\pi f} + \frac{a_2}{\alpha_2+j2\pi f}}$

This can be simplified more as follows.

$$S_Z(f) = \frac{a_1(\alpha_1+j2\pi f)+a_1(\alpha_1-j2\pi f)}{(\alpha_1-j2\pi f)(\alpha_1+j2\pi f)} + \frac{a_2(\alpha_2+j2\pi f)+a_2(\alpha_2-j2\pi f)}{(\alpha_2-j2\pi f)(\alpha_2+j2\pi f)}$$

$$= \boxed{\frac{2a_1\alpha_1}{\alpha_1^2+4\pi^2f^2} + \frac{2a_2\alpha_2}{\alpha_2^2+4\pi^2f^2}}$$