HW 11
Electronic Communication Systems
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California State University, Fullerson

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Contents

This was not collected. Practice problems for class only.

1 Problems

Find Hand out
do 4 problems
for sampling

high noise peaks. It is apparent that these false pulses have a finite though small probability of occurrence when the noise is Gaussian, no matter how small its standard deviation is compared with the peak amplitude of the pulses. As the transmission bandwidth is increased indefinitely, the accompanying increase in average noise power eventually causes the false pulses to occur often enough, thereby causing loss of the wanted message signal at the receiver output. *We thus find, in practice, that both PPM and PDM systems suffer from a threshold effect similar to that experienced in FM systems.*

Synchronization in Pulse-Time Modulation Systems

As with PAM systems, synchronization in pulse-time modulation systems is established by transmitting a distinctive marker per frame. In a PDM system, the marker may be identified by omitting a pulse, as illustrated in Fig. 7.13(*c*) for a PDM system involving three independent message sources. One method of identifying such a marker in the receiver is to utilize the charging time of a simple resistor-capacitor circuit to measure the duration of the intervals between duration-modulated pulses. The time constant of the circuit is chosen so that, during a marker interval, the voltage across the capacitor rises to a value considerably higher than that during the normal charging interval. Thus, by applying the output of the circuit to a slicer with an appropriate slicing level, the presence of a marker is detected.

In a PPM system, the marker pulse may be identified by making its duration several times longer than that of the message pulses, as illustrated in Fig. 7.13(*d*). At the receiver, the marker pulses may be separated from the message pulses by using a procedure essentially similar to that described for the PDM system. In this case, however, the capacitor is charged during the time of occurrence of each pulse, and discharged during the intervening intervals. Accordingly, the voltage across the capacitor reaches its highest value during the presence of a marker pulse, and the marker pulses are thereby separated from the message pulses.

Problems

✓ ***Problem 7.1*** The signal

$$g(t) = 10\cos(20\pi t)\cos(200\pi t)$$

is sampled at the rate of 250 samples per second.

(a) Determine the spectrum of the resulting sampled signal.
(b) Specify the cutoff frequency of the ideal reconstruction filter so as to recover $g(t)$ from its sampled version.
(c) What is the Nyquist rate for $g(t)$?
(d) By treating $g(t)$ as a band-pass signal, determine the lowest permissible sampling rate for this signal.

Problem 7.2 The signals

$$g_1(t) = 10\cos(100\pi t)$$

and

$$g_2(t) = 10\cos(50\pi t)$$

are both sampled at the rate of 75 samples per second. Show that the two sequences of samples thus obtained are identical. What is the reason for this phenomenon?

Problem 7.3 The signal

$$g(t) = 10\cos(60\pi t)\cos^2(160\pi t)$$

is sampled at the rate of 400 samples per second. Determine the range of permissible cutoff frequencies for the ideal reconstruction filter that may be used to recover $g(t)$ from its sampled version.

Problem 7.4 A signal $g(t)$ consists of two frequency components $f_1 = 3.9$ kHz and $f_2 = 4.1$ kHz in such a relationship that they just cancel each other out when the signal $g(t)$ is sampled at the instants $t = 0, T, 2T, \ldots$, where $T = 125\ \mu s$. The signal $g(t)$ is defined by

$$g(t) = \cos\left(2\pi f_1 t + \frac{\pi}{2}\right) + A\cos(2\pi f_2 t + \phi)$$

Find the values of amplitude A and phase ϕ of the second frequency component.

Problem 7.5 Let E denote the energy of a strictly band-limited signal $g(t)$. Show that E may be expressed in terms of the sample values of $g(t)$, taken at the Nyquist rate, as follows

$$E = \frac{1}{2W}\sum_{n=-\infty}^{\infty}\left|g\left(\frac{n}{2W}\right)\right|^2$$

where W is the highest frequency component of $g(t)$.

Problem 7.6 The spectrum of a signal $g(t)$ is shown in Fig. P7.1. This signal is sampled at the Nyquist rate with a periodic train of rectangular pulses of duration 50/3 milliseconds. Plot the spectrum of the sampled signal for frequencies up to 50 hertz.

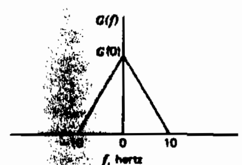

Figure P7.1

Problem 7.7 This problem is aimed at investigating the fact that practical electronic switching circuits will not produce a sampling function that consists of exactly rectangular pulses. Let $h(t)$ denote some arbitrary pulse shape, so that the sampling function $c(t)$ may be expressed as

$$c(t) = \sum_{n=-\infty}^{\infty} h(t - nT_s)$$

where T_s is the sampling period. The sampled version of an incoming analog signal $g(t)$ is defined by

$$s(t) = c(t)g(t)$$

(a) Show that the Fourier transform of $s(t)$ is given by

$$S(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} G\left(f - \frac{n}{T_s}\right)H\left(\frac{n}{T_s}\right)$$

where $G(f) = F[g(t)]$ and $H(f) = F[h(t)]$.

(b) What is the effect of using the arbitrary pulse shape $h(t)$?

Problem 7.8 Consider a continuous-time signal $g(t)$ of finite energy, with a continuous spectrum $G(f)$. Assume that $G(f)$ is sampled uniformly at the discrete frequencies $f = kF_s$, thereby obtaining the sequence of frequency samples $G(kF_s)$, where k is an integer in the entire range $-\infty < k < \infty$, and F_s is the frequency sampling interval. Show that if $g(t)$ is duration-limited, so that it is zero outside the interval $-T \leq t \leq T$, then the signal is completely defined by specifying $G(f)$ at frequencies spaced $1/2T$ hertz apart.

Problem 7.9

(a) Consider a stationary process $X(t)$ that is *not* strictly band-limited in the band W; that is,

$$S_X(f) \neq 0, \qquad |f| > W$$

where $S_X(f)$ is the power spectral density of the process. The process $X(t)$ is applied to an ideal low-pass filter defined by the transfer function

$$H(f) = \begin{cases} 1, & |f| < W \\ 0, & |f| > W \end{cases}$$

producing the process $X_f(t)$. This process is next sampled at a rate equal to $2W$, producing the sequence of samples $X_f(n/2W)$. An approximate reconstruction of the original process is defined by

$$Y(t) = \sum_{n=-\infty}^{\infty} X_f\left(\frac{n}{2W}\right)\operatorname{sinc}\left[2W\left(t - \frac{n}{2W}\right)\right]$$

Show that the mean-square value of the *sampling error* is

$$\begin{aligned}\mathscr{E} &= E[(X(t) - Y(t))^2] \\ &= 2\int_W^{\infty} S_X(f)\,df\end{aligned}$$

(b) Given that

$$S_X(f) = \frac{f_0}{f^2 + f_0^2}$$

determine the corresponding value of the mean-square error $\mathscr{E}$, and plot it as a function of W/f_0.

Problem 7.10 Consider a sequence of samples $x(nT_s)$ obtained by sampling a continuous-time signal $x(t)$ at the rate $1/T_s$. It is required to increase the sampling period T_s to a new value

2 Key solution

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Chapter 7

Pulse-Analog Modulation

Problem 7.1

(a) The signal g(t) is

$$g(t) = 10\cos(20\pi t)\cos(200\pi t)$$

$$= 5[\cos(220\pi t) + \cos(180\pi t)]$$

The Fourier transform of g(t) is

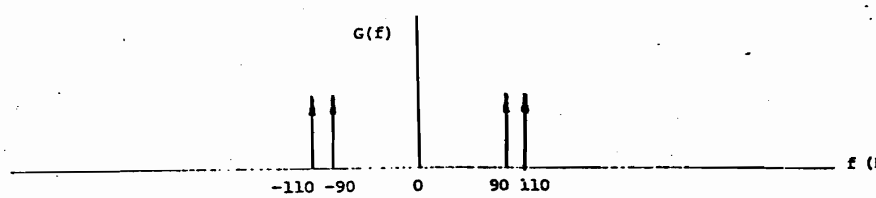
$$G(f) = 2.5[\delta(f-110) + \delta(f+110) + \delta(f-90) + \delta(f+90)]$$

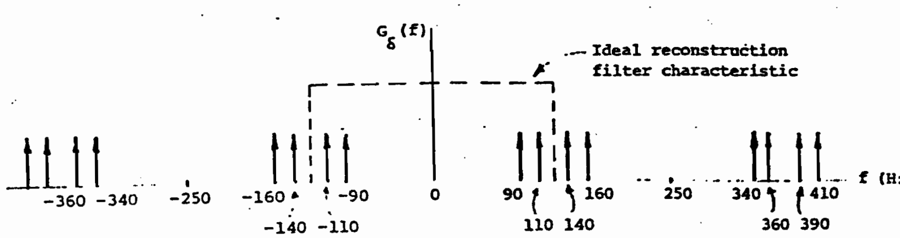
Hence, the spectrum of the sampled version of g(t), with a sampling period $T_s = 1/250$ s, is given by

$$G_\delta(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} G\left(f - \frac{n}{T_s}\right) = f_s \sum_{m=-\infty}^{+\infty} G(f - m f_s)$$

$$= 250 \times 2.5 \sum_{n=-\infty}^{\infty} [\delta(f-110-250n) + \delta(f+110-250n) + \delta(f-90-250n) + \delta(f+90-250n)]$$

(b) The spectra G(f) and $G_\delta(f)$ are illustrated below:





From this diagram, we deduce that in order to recover the original signal $g(t)$ from $g_\delta(t)$, we need to use a low-pass filter with a cutoff frequency that is greater than 110 Hz but less than 140 Hz.

(c) The highest frequency component of $g(t)$ is 110 Hz. The Nyquist rate of $g(t)$ is therefore 220 Hz.

(d) The signal $g(t)$ may be viewed as a band-pass signal occupying the frequency interval 90 to 110 Hz, that is,

$$f_u = 110 \qquad W = 110 - 90 = 20$$

$$f_s = \frac{2f_u}{m}$$

$$m \leq \frac{f_u}{W} = \frac{110}{20} = 5.5 \implies m = 5$$

$$f_s = \frac{2f_u}{m} = \frac{2 \times 110}{5} = 44\,Hz.$$

Problem 7.2

The spectrum of $g_1(t)$ is

$$G_1(f) = 5[\delta(f-50) + \delta(f+50)]$$

Hence, the spectrum of the sampled version of $g_1(t)$, using a sampling period $T_s = 1/75$ s, is

$$G_{1\delta}(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} G_1(f - \frac{n}{T_s})$$

$$= \cancel{1}375 \sum_{n=-\infty}^{\infty} [\delta(f-50-75n) + \delta(f+50-75n)] \qquad (1)$$

Next, the spectrum of $g_2(t)$ is

$$G_2(f) = 5[\delta(f-25) + \delta(f+25)]$$

Hence, the spectrum of the sample version of $g_2(t)$, using a sampling period T_s =1/75 s, is

$$G_{2\delta}(f) = 375 \sum_{n=-\infty}^{\infty} [\delta(f-25-75n) + \delta(f+25-75n)] \qquad (2)$$

In the right-hand side of Eq. (2), substitute n = ℓ-1 for the first term, and n = m+1 for the second term, and so rewrite this equation as follows:

$$G_{2\delta}(f) = 375 \sum_{\ell=-\infty}^{\infty} \delta(f+50-75\ell) + 375 \sum_{m=-\infty}^{\infty} \delta(f-50-75m)$$

$$= 375 \sum_{n=-\infty}^{\infty} [\delta(f-50-75n) + \delta(f+50-75n)] \qquad (3)$$

We thus find from Eqs. (1) and (3) that the spectra $G_{1\delta}(f)$ and $G_{2\delta}(f)$ are identical. That is, the sample versions of $g_1(t)$ and $g_2(t)$ are identical.

We note that the Nyquist rate of $g_1(t)$ is 100 Hz; hence, with a sampling rate of 75 Hz, the signal $g_1(t)$ is under-sampled by 25 Hz below the Nyquist rate. On the other hand, the Nyquist rate of $g_2(t)$ is 50 Hz; hence, the signal $g_2(t)$ is over-sampled by 25 Hz above the Nyquist rate. Thus, although $g_1(t)$ and $g_2(t)$ represent two sinusoidal waves of different frequencies, we find that by under-sampling $g_1(t)$ and over-sampling $g_2(t)$ appropriately, their sampled versions are identical.

Problem 7.3

Express the signal g(t) as

$$g(t) = 10\cos(60\pi t)\cos^2(160\pi t)$$

$$= 5\cos(60\pi t)[1 + \cos(320\pi t)]$$

$$= 5\cos(60\pi t) + 2.5\cos(380\pi t) + 2.5\cos(260\pi t)$$

The spectrum of g(t) is

$$G(f) = 2.5[\delta(f-30) + \delta(f+30)] + 1.25[\delta(f-190) + \delta(f+190)] + 1.25[\delta(f-130) + \delta(f+130)]$$

The corresponding spectrum of the sampled versionof g(t), using a sampling rate of 400 Hz, is therefore

$$G_\delta(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} G(f - \frac{n}{T_s})$$

$$= 400 \sum_{n=-\infty}^{\infty} \begin{bmatrix} 2.5[\delta(f-30-400n) + \delta(f+30-400n)] \\ + 1.25[\delta(f-190-400n) + \delta(f+190-400n)] \\ + 1.25[\delta(f-130-400n) + \delta(f+130-400n)] \end{bmatrix}$$

The spectra G(f) and $G_\delta(f)$ are illustrated below:

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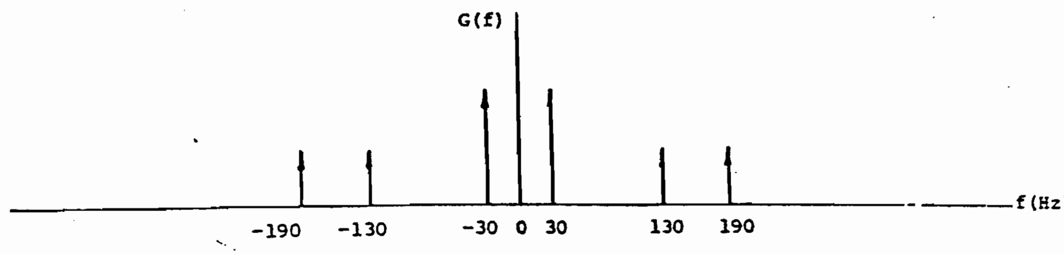

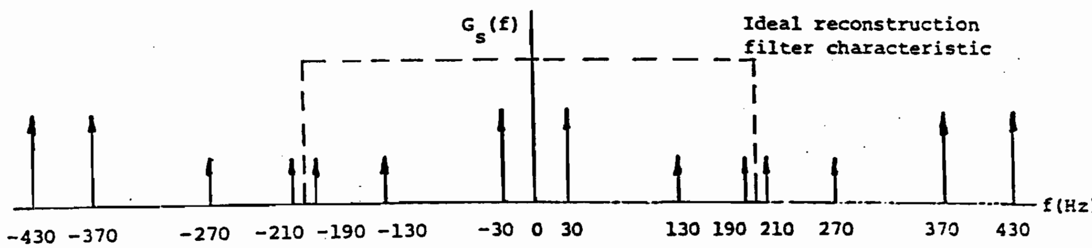

From this diagram, we deduce that in order to recover the original signal $g(t)$ from its sampled version, the low-pass reconstruction filter must have a cutoff frequency greater than 190 Hz but less than 210 Hz.

Problem 7.4

The signal at the sampling instants is:

$$g(nT) = \cos(2\pi f_1 nT + \frac{\pi}{2}) + A\cos(2\pi f_2 nT + \phi)$$

$$= 0\,, \qquad n = 0, 1, 2, \ldots$$

At $n = 0$,

$$\cos(\frac{\pi}{2}) + A\cos\phi = 0\,. \tag{1}$$

At $n = 1, 2, \ldots$, with $f_1 = 3.9$ kHz, $f_2 = 4.1$ kHz, and $T = 125$ μs, we have

$$\cos(0.975n\pi + \frac{\pi}{2}) + A\cos(1.025\,n\pi + \phi) = 0. \tag{2}$$

From (2) and $\cos(0.975n\pi + \frac{\pi}{2})$ being non-zero, A must be non-zero. From (1) and A being non-zero, ϕ must be $\pm\frac{\pi}{2}$. Equation (2) then becomes:

$$-\sin(0.975n\pi) \pm A\sin(1.025n\pi) = 0. \tag{3}$$

Since $\sin(\cdot)$ is odd symmetric about $n\pi$, A equals 1 and the ambiguous sign in (3) is negative. Therefore, $\phi = \frac{\pi}{2}$.